

# Analysis of Billet Rolling in a Continuous Mill using Idle Vertical Stands

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An analytical approach is presented to investigate the deformation characteristics of billets in a continuous billet mill using power driven horizontal stands and idle vertical stands. The analysis is validated by comparison to the experimental results in a previously published work. The analytical results have shown that, apart from the problems of slip and buckling of billet, there are some shortcomings involved in this method. Compared to conventional rolling with all driven stands, the roll load for idle vertical stands and the rolling torque for horizontal stands are almost doubled. The billet is severely stressed within the roll-bite of idle vertical stands and the overall rolling power has increased by one third of that for conventional rolling. These shortcomings impair the feasibility of industrial application of idle vertical stand rolling method.

**Key Words :** Metal Hot Working, Billet Rolling, Idle Stands, Roll Pass Design

## Nomenclature

$A$	: Work piece cross sectional area
$F$	: Interstand tension or compression
$L_r$	: Roll-bite length
$L_s$	: Distance between two successive stands
$M$	: Rolling torque
$N$	: Rolling power
$P$	: Roll load
$R$	: Roll radius
$V_o, V_e$	: Entry and exit pass velocity
$2w$	: Billet width
$2h$	: Billet height
$v_x, v_y, v_z$	: Billet material velocity components
$\lambda$	: Interstand velocity change ratio
$\sigma_o, \sigma_e$	: Back and front axial stress

$\omega$  : Roll angular velocity

## Subscripts

$o$	: At roll-bite entry
$e$	: At roll-bite exit
$r$	: Along roll-bite length
$s$	: Along interstand length

## 1. Introduction

In the roughing train of a bar mill, square or rectangular billets are broken down by being rolled in power-driven horizontal and vertical stands sequence. Power-driven horizontal stands are compact and have simple mechanical structures whereas power-driven vertical stands have greater height and complicated structures that results in quite higher investment and maintenance costs. In an attempt to reduce cost in the vertical stands it was proposed to eliminate the power drive system in these stands so that the billet mill consists of power-driven horizontal stands and idle vertical stands using grooveless

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(flat) rolls. In this type of rolling, the billet is pushed into the roll gap of the vertical idle rolls by the grip of the preceding power-driven horizontal stand and is simultaneously pulled out by the grip of the power-driven horizontal stand following the vertical idle rolls.

Under such rolling conditions two problems are involved; namely excessive slip in the roll gap of the horizontal stand and the tendency of the billet length between the vertical idle rolls and the preceding pushing horizontal rolls to buckle. These two problems were thoroughly investigated experimentally in a previously published work by Shikano et al. (1991). They carried out tests using different billet sizes and area reduction ratios on a specially constructed 5-stand continuous rolling mill with idle vertical stands (Patent, 0000). Measurements for the roll load and torque and the interstand forces were carried out. The results obtained have shown that slip depended upon the amount of thickness reduction by the idle rolls in relation to the thickness reduction in the driven rolls. The tendency for buckling was related to the thickness reduction in the idle rolls, the distance between stands and the billet thickness. Limiting conditions to avoid slip and buckling were experimentally determined from the tests. Based on their experimental results they concluded that the method was promising for industrial application. In order to proceed towards industrial application the method needs to be theoretically investigated, to predict its characteristics under various rolling conditions.

In the present work an analytical investigation to predict the roll load and rolling torque in continuous billet rolling process using idle vertical stands is presented. The investigation is also intended to determine the variation of billet dimensions along the interstand distance and to predict the limiting conditions to avoid excessive slip and buckling of billet. While several analytical and finite element investigations to predict the characteristics of conventional grooveless billet rolling with all driven stands are available in the literature (Yanazawa et al., 1983; Kandaurov et al., 1989; Oh and Kobayashi, 1975; Kennedy, 1987; Park and Oh, 1990; Mori

and Osakada, 1990; Yanagimoto and Kiuchi, 1991; Komori, 2002; Laila et al., 2004), theoretical investigations dealing with the rolling of billets using idle vertical stands have not been attempted. The approach proposed in the present analysis is based on the adaptation of a previous work by the authors on bar rolling (Laila et al., 2004). The results obtained from the analysis are compared with the experimental results of Ref. (Shikano et al., 1991) and to the results of conventional billet rolling with all-driven stands.

## 2. Problem Formulation

The analysis is based on assuming homogeneous deformation through the billet height and width and uniform rolling velocity across the billet cross section. The billet material is considered to be isotropic, rigid-plastic and incompressible. The flow stress of the material is obtained by substituting the effective strain and effective strain rate into the material constitutive law at the prevailing rolling temperature. The deformation is isothermal and roll flattening is assumed to be negligible during rolling.

### 2.1 Roll-bite geometry

Referring to Fig. 1, Cartesian coordinates  $x$ ,  $y$ ,  $z$  are chosen to be the directions of billet length, height and width respectively, with the origin of axes at the midpoint of the entry plane. The  $x$ - $y$  and  $x$ - $z$  planes are planes of symmetry.  $2h_o$ ,  $2h_e$ ,  $2h$  are the cross section height,  $2w_o$ ,  $2w_e$ ,  $2w$  are the cross section width and  $A_o$ ,  $A_e$ ,  $A$  are the cross sectional area at entry, exit and any cross section along the roll-bite length, respectively, where  $A_o=4h_o w_o$ ,  $A_e=4h_e w_e$  and  $A=4h w$ . It is presumed that the billet cross sectional dimensions at pass entry and exit are already known.

The variation of section height  $2h$  at any distance  $x$  from entry is given by

$$h = h_o - \frac{L_r}{R} x + \frac{x^2}{2R} \quad (1)$$

which satisfies the boundary conditions of  $h=h_o$  at  $x=0$  and  $h=h_e$  at  $x=L_r$ .

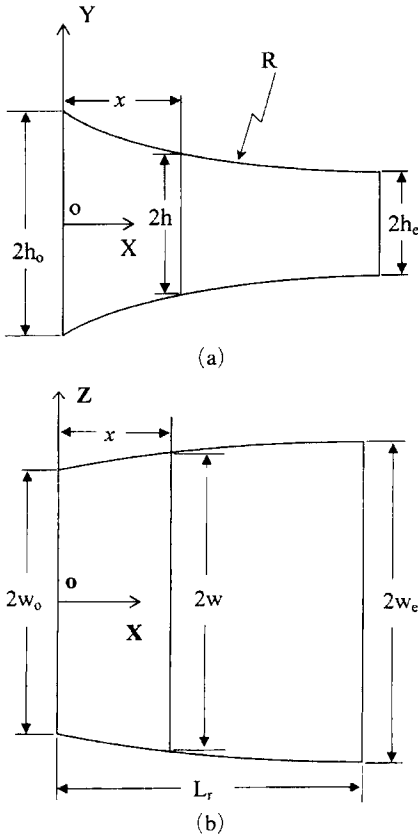


Fig. 1 Roll-bite geometry

$R$  is the roll radius and  $L_r$  is the projected roll-bite length obtained as

$$L_r = \sqrt{2R(h_o - h_e)} \tag{2}$$

The section width  $2w$  is assumed to increase along the roll-bite length into a parabolic shape as

$$w = w_o - 2(w_o - w_e) \frac{x}{L_r} + (w_o - w_e) \frac{x^2}{L_r^2} \tag{3}$$

which satisfies the boundary conditions of  $w = w_o$  at  $x = 0$ , and  $w = w_e$  and  $dw/dx = 0$  at  $x = L_r$ .

**2.2 Velocity, strain and strain rate components**

a) Along roll-bite length

The billet enters the roll-bite with an axial velocity  $V_o = V_e A_e / A_o$  and exits the roll-bite with an axial velocity  $V_e = \omega R$ , where  $\omega$  is the angular velocity of the rolls. The axial velocity

component  $v_x$  at any cross section of distance  $x$  from entry is assumed to be uniform across the billet cross section and is obtained from the constant volume flow condition as

$$v_x = \frac{V_e h_e w_e}{h_w} \tag{4}$$

Considering the deformation of the billet cross section to be homogeneous in the directions of height and width, the respective strain components are given by

$$\epsilon_y = \ln \frac{h}{h_o}, \quad \epsilon_z = \ln \frac{w}{w_o} \tag{5}$$

and from the constant volume condition

$$\epsilon_x = -(\epsilon_y + \epsilon_z) \tag{6}$$

Neglecting the shear strain components, the effective strain is obtained as

$$\bar{\epsilon}_r = \sqrt{\frac{2}{3}(\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2)} \tag{7}$$

The strain rate components are obtained by differentiating the strain components with respect to time

$$\dot{\epsilon}_y = \frac{v_x}{h} \frac{\partial h}{\partial x}, \quad \dot{\epsilon}_z = \frac{v_x}{w} \frac{\partial w}{\partial x}, \quad \dot{\epsilon}_x = -(\dot{\epsilon}_y + \dot{\epsilon}_z) \tag{8a}$$

$$\dot{\bar{\epsilon}}_r = \sqrt{\frac{2}{3}(\dot{\epsilon}_x^2 + \dot{\epsilon}_y^2 + \dot{\epsilon}_z^2)} \tag{8b}$$

b) Along interstand length

The billet emerges from the roll-bite with a velocity  $V_e$  which due to interstand tension or compression will respectively increase or decrease along the interstand length  $L_s$  to reach the entry of the next stand at a velocity  $V_o = (1 + \lambda) V_e$ , where  $\lambda$  is the ratio of the change in billet velocity throughout the interstand length to the exit velocity  $V_e$ . The state of stress in the billet along the interstand length is a uniaxial stress so that the strain components upon entry to the roll-bite of the next stand are expressed as

$$\epsilon_x = \bar{\epsilon}_s = \lambda \tag{9a}$$

$$\epsilon_y = \epsilon_z = -0.5 \epsilon_x \tag{9b}$$

The strains in Eqs. (9a) and (9b) are small such that the engineering and logarithmic strains are

almost the same. The strain rate components can be expressed as

$$\dot{\epsilon}_x = \dot{\epsilon}_s = \frac{\lambda V_e}{L_s}, \quad \dot{\epsilon}_y = \dot{\epsilon}_z = -0.5 \dot{\epsilon}_x \quad (10)$$

### 2.3 Stresses

#### a) Along roll-bite length

The flow stress is obtained by substituting the effective strain and effective strain rate, from Eqs. (7) and (8) into the material constitutive law at the rolling temperature. The deviatoric stress components  $\sigma'_x, \sigma'_y, \sigma'_z$  are obtained from Levy-Mises flow rule given by the relations

$$\sigma'_x = \frac{2\bar{\sigma}}{3\bar{\epsilon}_r} \dot{\epsilon}_x, \quad \sigma'_y = \frac{\sigma\bar{\sigma}}{3\bar{\epsilon}_r} \dot{\epsilon}_y, \quad \sigma'_z = \frac{2\bar{\sigma}}{3\bar{\epsilon}_r} \dot{\epsilon}_z \quad (11)$$

The mean stress  $\sigma_m$  is obtained from the stress component  $\sigma_x$  as

$$\sigma_m = \sigma_x - \sigma'_x \quad (12)$$

In conventional rolling with back and front stresses,  $\sigma_o, \sigma_e$  respectively,  $\sigma_x$  may be expressed by a parabolic relation along the roll-bite length as

$$\sigma_x = \sigma_o - 2(\sigma_o - \sigma_e) \frac{x}{L_r} + (\sigma_o - \sigma_e) \frac{x^2}{L_r^2} \quad (13)$$

which satisfies the conditions that :

$$\sigma_x = \sigma_o \text{ at } x=0 \text{ and } \sigma_x = \sigma_e \text{ at } x=L_r \quad (14)$$

The other normal stress components are obtained from Eqs. (12) and (13) as

$$\sigma_y = \sigma_x - \sigma'_x + \sigma'_y, \quad \sigma_z = \sigma_x - \sigma'_x + \sigma'_z \quad (15)$$

In the case of idle rolls subjected to front and back stresses, the distribution of  $\sigma_x$  along the roll-bite length should satisfy, apart from the two conditions of Eq. (14), the further condition of zero rolling torque. Eq. (13) is therefore arbitrarily modified by inserting a factor  $f$  which is to be determined from the further condition of zero rolling torque. The general expression for the variation of  $\sigma_x$  along the roll-bite length is thus given by the parabolic relation

$$\sigma_x = \sigma_o - (1+f)(\sigma_o - \sigma_e) \frac{x}{L_r} + f(\sigma_o - \sigma_e) \frac{x^2}{L_r^2} \quad (16)$$

in which  $f=1$  in the case of power-driven rolls. The back and front stresses,  $\sigma_o$  and  $\sigma_e$ , are to be determined from the conditions along the interstand length.

#### b) Along interstand length

The flow stress is obtained by substituting the effective strain and effective strain rate from Eqs. (9) and (10) into the material constitutive law at the rolling temperature. Since the state of stress along the interstand length is uniaxial then the stresses  $\sigma_o$  and  $\sigma_e$  are the flow stresses of the billet along the interstand length at the entry to and the exit from the roll-bite of the pass. The back and front tensile forces for the pass are obtained from the relations  $F_o = A_o \sigma_o$  and  $F_e = A_e \sigma_e$ .

### 2.4 Roll load, rolling torque and power

The rolling load  $P$ , rolling torque  $M$  and rolling power  $N$  are respectively

$$P = -2 \int_0^{L_r} \sigma_y w \, dx \quad (17a)$$

$$M = -4 \int_0^{L_r} \sigma_y w (L_r - x) \, dx + R(F_o - F_e) \quad (17b)$$

$$N = M\omega \quad (17c)$$

where  $\omega$  is the roll angular velocity.

### 2.5 Billet dimensions

It is presumed that the billet dimensions are already known in case of no back and front tensions. An increase or decrease in the exit velocity  $V_e$  by a ratio of  $\lambda$  along the interstand length will decrease or increase the entry billet cross sectional area to the next pass by  $1/(1+\lambda)$ . This leads to a decrease or increase of the cross sectional area at the exit of the pass by the same ratio. Since the exit billet height is determined by the roll gap, which is maintained unchanged, then the reduction will occur in only the billet width  $2w_e$  to become equal to  $2w_e/(1+\lambda)$ .

### 2.6 Slip in horizontal stands

It has been shown from Eqs. (9) and (10) that increasing  $\lambda$  will increase the effective strain and strain rate along the interstand length which in

turn increases the interstand stress. Assume now that  $\lambda$  is increased to make the back stress in a horizontal stand equal to the flow stress upon entry to the roll-bite. In this case there will be no stress discontinuity between the interstand length and the entry plane so that the flow stress of the billet material becomes equal to  $\sigma_x$  at the entry to the roll-bite. This means that the other two stress components  $\sigma_y$  and  $\sigma_z$  do not prevail, which then leads to the loss of the ability of the work rolls to grip the workpiece and excessive slip along the roll bite.

### 2.7 Billet buckling

The problem of billet buckling due to interstand compression upon entering the roll gap of the idle vertical stands has been thoroughly investigated experimentally in Ref. (Shikano et al., 1991). The results have shown that for the billets used in the tests the buckling stress  $\sigma_{\text{buckling}}$  was expressed as the linear relation

$$\sigma_{\text{buckling}} = 35(1 - 0.014S_r) \quad (18)$$

where  $S_r$  is the slenderness ratio of the billet cross section considering the billet as a beam fixed from one end by the horizontal rolls and simply supported at the other end by the idle vertical rolls. In this case the effective buckling length is  $0.7L_r$  and the slenderness ratio is expressed as

$$S_r = 1.2L_r/h \quad (19)$$

## 3. Results and Discussion

The results obtained from the analysis are compared with the experimental results in Table 1. The billet material is 0.2% plain carbon steel rolled at 1100°C. The flow stress is expressed by Shida's constitutive equation (Shida, 1969) as

$$\bar{\sigma} = 111.2 [1.639(\bar{\epsilon})^{0.396} - \bar{\epsilon}] (\dot{\bar{\epsilon}})^{0.133} \quad (20)$$

The billet was rolled in a five-stands horizontal/vertical continuous mill with power driven horizontal stands and idle vertical stands. The roll diameters are 300 mm for the horizontal stands and 250 mm for the vertical stands. The distance

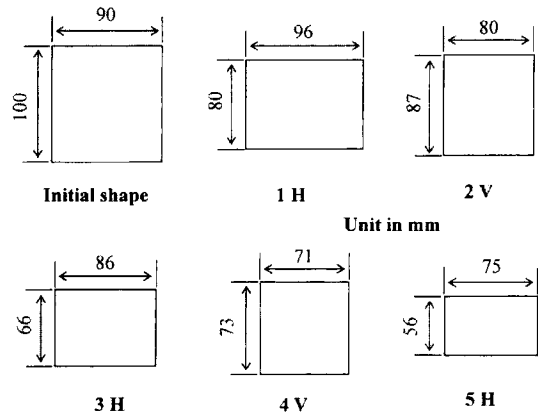


Fig. 2 Roll pass schedule

between two successive stands is 715 mm for the first three stands and 850 mm between the last two stands. Fig. 2 shows the roll pass schedule which was used in the tests.

Tables 1 shows the passes data and experimental results as obtained from Ref. (Shikano et al., 1991).

The theoretical results obtained from the proposed analysis for the same experimental data are given in Table 2.

Comparing the results of the theoretical and experimental values of the roll load and rolling torque in Tables 1 and 2 shows that the deviations in the roll load for stands 1, 2, 3, 4 and 5 are -3.9%, 11.25, -1.6%, 10% and -8% respectively and for the rolling torque for the horizontal stands 1, 3 and 5 are -13%, 10% and 5% respectively which indicates a fairly agreement.

The results also show that the back stress is almost 50% of the entry flow stress which means that excessive slip is eliminated. The buckling stress between stands 1 and 2 and stands 3 and 4 is more than double the interstand compressive stress which means that buckling is also eliminated. The compressive and tensile interstand strains,  $\lambda$ , are quite small so that the change in billet dimensions may be neglected.

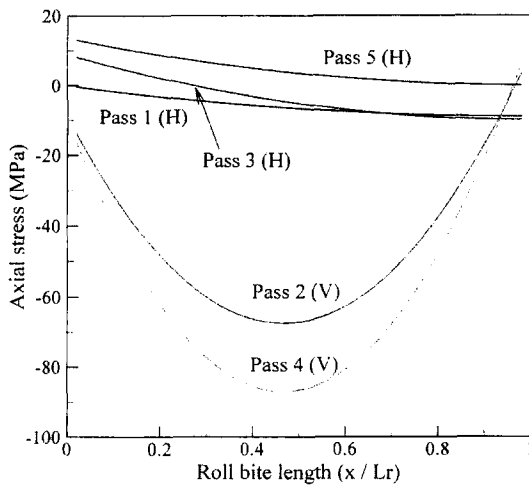
Figs. 3 and 4 show respectively the theoretical distribution of the axial stress and roll pressure along the roll-bite length for the five passes with power-driven horizontal and idle vertical stands.

**Table 1** Passes data and experimental results (Shikano et al., 1991)

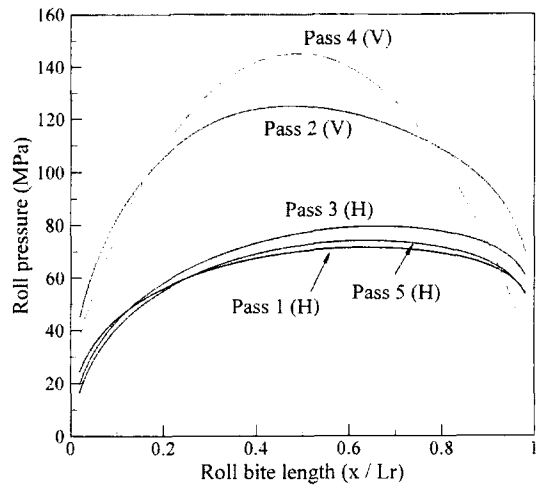
Pass No	1	2	3	4	5
Roll axis	Horizontal	Vertical	Horizontal	Vertical	Horizontal
Exit speed $V_e$ (m/s)	0.044	0.049	0.061	0.065	0.084
Back stress $\sigma_o$ (MPa)	0	-9	8.9	-10	13.6
Front stress $\sigma_e$ (MPa)	-9	8.9	-10	13.6	0
Roll Load $P$ (kN)	330	320	320	300	250
Roll Torque $M$ (kNm)	30	0	30	0	20
Rolling power $N$ (kW)	8.8	0	12.2	0	10.7
Total rolling power=31.7 (kW)					

**Table 2** Theoretical results from the proposed analysis

Pass No	1	2	3	4	5
Roll axis	Horizontal	Vertical	Horizontal	Vertical	Horizontal
Entry flow stress (Mpa)	17	16	19.4	17.26	20.24
Buckling stress (Mpa)		-24.5	0	-22.26	0
Interstand axial strain ( $\lambda$ )		-0.0073	0.007	-0.0083	0.0142
Roll-bite length $L_r$ (mm)	54.77	44.72	56.12	43.3	50.5
Stress factor ( $f$ )	1	-15	1	-15	1
Roll Load $P$ (kN)	317	356	315	330	230
Roll Torque $M$ (kNm)	26	0	33	0	21



**Fig. 3** Distribution of axial stress along roll-bite length for driven horizontal and idle vertical stands



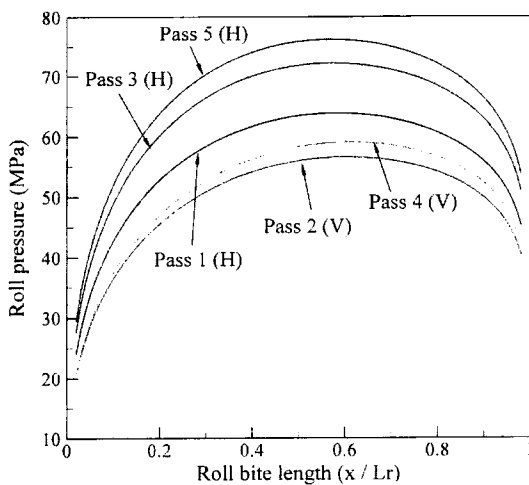
**Fig. 4** Distribution of roll pressure along roll-bite length for driven horizontal and idle vertical stands

The curves indicate that the axial stress and roll pressure for the idle vertical rolls are much higher compared with those for the driven horizontal

stands. This means that using drive-free vertical stands will subject the billet to excessive stresses along the roll-bite length.

**Table 3** Theoretical results for conventional rolling

Pass No	1	2	3	4	5
Roll axis	Horizontal	Vertical	Horizontal	Vertical	Horizontal
Roll Load $P_{th}$ (kN)	286	187	296	155	247
Roll Load $P_{exp}$ (kN)	270	—	—	—	250
Roll Torque $M_{th}$ (kNm)	15	8	16	6.25	12
Roll Torque $M_{exp}$ (kNm)	16	—	—	—	11
Torque arm ratio ( $M/2PL_r$ )	0.479	0.478	0.482	0.466	0.481
Rolling power $N$ (kW)	4.4	3.12	6.45	3.26	6.43
Total rolling power=23.66 (kW)					

**Fig. 5** Distribution of roll pressure along the roll-bite length for conventional rolling with all-driven stands

The results obtained for conventional billet rolling, in which all stands are power driven with no interstand stresses, are shown in Table 3 and the distribution of pressure along roll bite length is shown in Fig. 5.

Comparing the results of Table 1 to those of Table 3 indicates that for the horizontal stands there is no appreciable change in the roll load whereas the rolling torque using idle vertical stands is nearly doubled.

For the vertical stands the roll load using idle rolls is nearly double that for conventional rolling. The rolling torque for driven vertical stands in conventional rolling is almost 50% of that for horizontal stands. The total rolling power for the five passes when using idle vertical rolls is

1.33 times of that for conventional rolling.

#### 4. Conclusions

Billet rolling in a continuous mill using idle vertical stands has the advantages of a compact and drive-free economical mill. However, the results obtained from the analysis have shown that, apart from the problems of slip in driven horizontal stands and billet buckling along the interstand length, there exist some shortcomings. Compared to conventional rolling with all driven stands, the roll load for idle vertical stands and the rolling torque for horizontal stands are almost doubled. The billet is severely stressed in the roll-bite of idle vertical stands and the overall rolling power has increased to 1.33 of that for conventional rolling. These shortcomings impair the feasibility of the industrial application of idle vertical stand rolling technology.

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